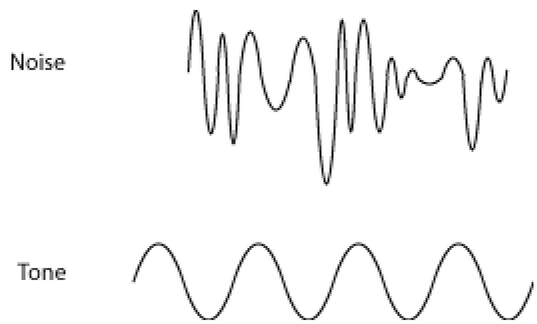
Acoustics for Music Theory
For adults, a short review of the physics underlying music theory.

Music is Organized Sound Waves

Music is sound that's organized by people on purpose, to dance to, to tell a story, to make other people feel a certain way, or just to sound pretty or be entertaining. Music is organized on many different levels. Sounds can be arranged into melodies, harmonies, rhythms, textures and phrases. Beats, measures, cadences, and form all help to keep the music organized and understandable. But the most basic way that music is organized is by arranging the actual sound waves themselves so that the sounds are interesting and pleasant and go well together.

A rhythmic, organized set of thuds and crashes is perfectly good music - think of your favorite drum solo - but many musical instruments are designed specifically to produce the regular, evenly spaced sound waves that we hear as particular pitches. Crashes, thuds, and bangs are loud, short jumbles of lots of different wavelengths. These are the kinds of sound we often call "noise", when they're random and disorganized, but as soon as they are organized in time (rhythm), they begin to sound like music. (When used as a scientific term, **noise** refers to **continuous** sounds that are random mixtures of different wavelengths, not shorter crashes and thuds.)

However, to get the melodic kind of sounds more often associated with music, the sound waves must themselves be organized and regular, not random mixtures. Most of the sounds we hear are brought to our ears through the air. A movement of an object causes a disturbance of the normal motion of the air molecules near the object. Those molecules in turn disturb other nearby molecules out of their normal patterns of random motion, so that the disturbance itself becomes a thing that moves through the air - a sound wave. If the movement of the object is a fast, regular vibration, then the sound waves are also very regular. We hear such regular sound waves as **tones**, sounds with a particular pitch. It is this kind of sound that we most often associate with music, and that many musical instruments are designed to make.



A random jumble of sound waves is heard as a noise. A regular, evenly-spaced sound wave is heard as a tone.

Musicians have terms that they use to describe tones. (Musicians also have other meanings for the word "tone", but this course will stick to the "a sound with pitch" meaning.) This kind of (regular, evenly spaced) wave is useful for things other than music, however, so scientists and engineers also have terms that describe pitched sound waves. As we talk about where music theory comes from, it will be very useful to know both the scientific and the musical terms and how they are related to each other.

For example, the closer together those evenly-spaced waves are, the higher the note sounds. Musicians talk about the pitch of the sound, or name specific notes, or talk about tuning. Scientists and engineers, on the other hand, talk about the frequency and the wavelength of the sound. They are all essentially talking about the same things, but talking about them in slightly different ways, and using the scientific ideas of wavelength and frequency can help clarify some of the main ideas underlying music theory.

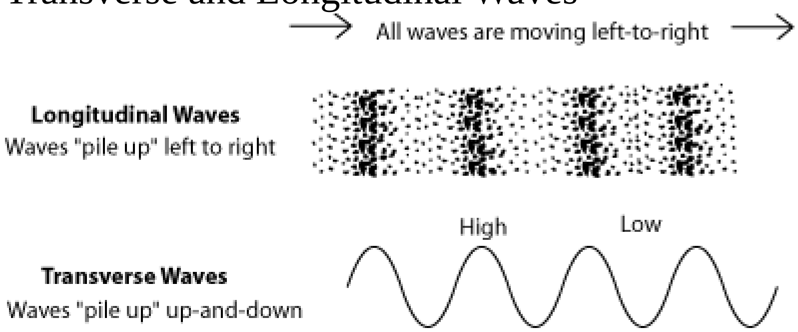
Longitudinal and Transverse Waves

So what are we talking about when we speak of sound waves? Waves are disturbances; they are changes in something - the surface of the ocean, the air, electromagnetic fields. Normally, these changes are travelling (except

for standing waves); the disturbance is moving away from whatever created it, in a kind of domino effect.

Most kinds of waves are **transverse** waves. In a transverse wave, as the wave is moving in one direction, it is creating a disturbance in a different direction. The most familiar example of this is waves on the surface of water. As the wave travels in one direction - say south - it is creating an up-and-down (not north-and-south) motion on the water's surface. This kind of wave is fairly easy to draw; a line going from left-to-right has up-and-down wiggles. (See [link].)

Transverse and Longitudinal Waves



In water waves and other **transverse waves**, the ups and downs are in a different direction from the forward movement of the wave. The "highs and lows" of sound waves and other **longitudinal waves** are arranged in the "forward" direction.

But sound waves are not transverse. Sound waves are **longitudinal waves**. If sound waves are moving south, the disturbance that they are creating is giving the air molecules extra north-and-south (not east-and-west, or up-and-down) motion. If the disturbance is from a regular vibration, the result is that the molecules end up squeezed together into evenly-spaced waves. This is very difficult to show clearly in a diagram, so **most diagrams, even diagrams of sound waves, show transverse waves**.

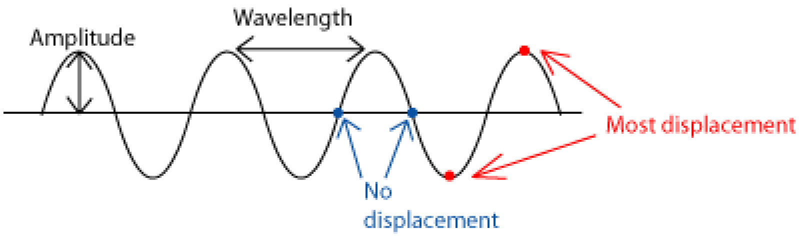
Longitudinal waves may also be a little difficult to imagine, because there aren't any examples that we can see in everyday life (unless you like to play with toy slinkies). A mathematical description might be that in longitudinal waves, the waves (the disturbances) are along the same axis as the direction of motion of the wave; transverse waves are at right angles to the direction of motion of the wave. If this doesn't help, try imagining yourself as one of the particles that the wave is disturbing (a water drop on the surface of the ocean, or an air molecule). As it comes from behind you, a transverse waves lifts you up and then drops down; a longitudinal wave coming from behind pushes you forward and pulls you back. You can view here animations of longitudinal and transverse waves, single particles being disturbed by a transverse wave or by a longitudinal wave, and particles being disturbed by transverse and longitudinal waves.

The result of these "forward and backward" waves is that the "high point" of a sound wave is where the air molecules are bunched together, and the "low point" is where there are fewer air molecules. In a pitched sound, these areas of bunched molecules are very evenly spaced. In fact, they are so even, that there are some very useful things we can measure and say about them. **In order to clearly show you what they are, most of the diagrams in this course will show sound waves as if they are transverse waves**.

Wave Amplitude and Loudness

Both transverse and longitudinal waves cause a **displacement** of something: air molecules, for example, or the surface of the ocean. The amount of displacement at any particular spot changes as the wave passes. If there is no wave, or if the spot is in the same state it would be in if there was no wave, there is no displacement. Displacement is biggest (furthest from "normal") at the highest and lowest points of the wave. In a sound wave, then, there is no displacement wherever the air molecules are at a normal density. The most displacement occurs wherever the molecules are the most crowded or least crowded.

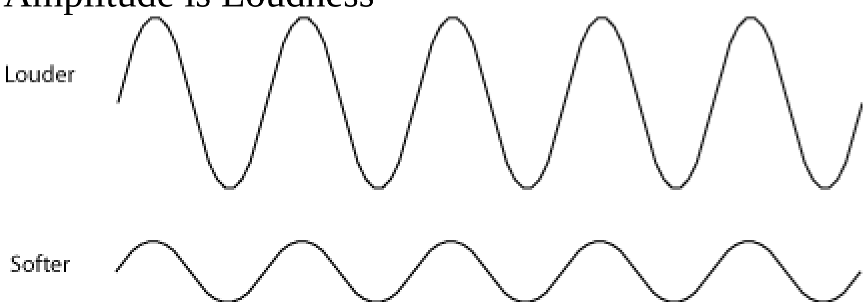
Displacement



The **amplitude** of the wave is a measure of the displacement: how big is the change from no displacement to the peak of a wave? Are the waves on the lake two inches high or two feet? Are the air molecules bunched very tightly together, with very empty spaces between the waves, or are they barely more organized than they would be in their normal course of bouncing off of each other? Scientists measure the amplitude of sound waves in **decibels**. Leaves rustling in the wind are about 10 decibels; a jet engine is about 120 decibels.

Musicians call the loudness of a note its **dynamic level**. **Forte** (pronounced "FOR-tay") is a loud dynamic level; **piano** is soft. Dynamic levels don't correspond to a measured decibel level. An orchestra playing "fortissimo" (which basically means "even louder than forte") is going to be quite a bit louder than a string quartet playing "fortissimo". (See Dynamics for more of the terms that musicians use to talk about loudness.) Dynamics are more of a performance issue than a music theory issue, so amplitude doesn't need much discussion here.

Amplitude is Loudness



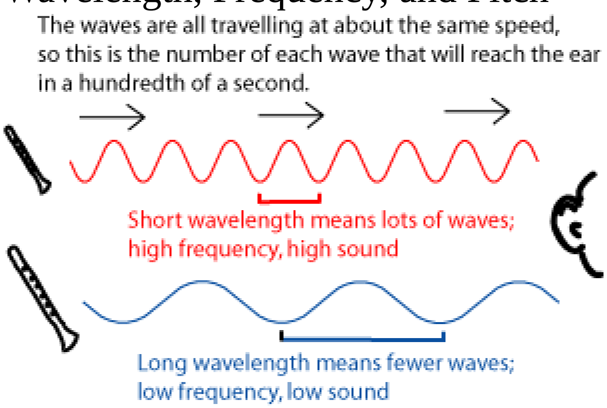
The size of a wave (how much it is "piled up" at the high points) is its **amplitude**. For sound waves, the bigger the amplitude, the louder the sound.

Wavelength, Frequency, and Pitch

The aspect of evenly-spaced sound waves that really affects music theory is the spacing between the waves, the distance between, for example, one high point and the next high point. This is the **wavelength**, and it affects the pitch of the sound; the closer together the waves are, the higher the tone sounds.

All sound waves are travelling at about the same speed - the speed of sound. So waves with a shorter wavelength arrive (at your ear, for example) more often (frequently) than longer waves. This aspect of a sound - how often a peak of a wave goes by, is called **frequency** by scientists and engineers. They measure it in **hertz**, which is how many peaks go by per second. People can hear sounds that range from about 20 to about 17,000 hertz.

Wavelength, Frequency, and Pitch



Since the sounds are travelling at about the same speed, the one with the shorter wavelength "waves" more frequently; it has a higher frequency, or pitch. In other words, it sounds higher.

The word that musicians use for frequency is **pitch**. The shorter the wavelength, the higher the frequency, and the higher the pitch, of the sound. In other words, short waves sound high; long waves sound low. Instead of

measuring frequencies, musicians name the pitches that they use most often. They might call a note "middle C" or "second line G" or "the F sharp in the bass clef". (See Octaves and Diatonic Music and Tuning Systems for more on naming specific frequencies.) These notes have frequencies (Have you heard of the "A 440" that is used as a tuning note?), but the actual frequency of a middle C can vary a little from one orchestra, piano, or performance, to another, so musicians usually find it more useful to talk about note names.

Most musicians cannot name the frequencies of any notes other than the tuning A (440 hertz). The human ear can easily distinguish two pitches that are only one hertz apart when it hears them both, but it is the very rare musician who can hear specifically that a note is 442 hertz rather than 440. So why should we bother talking about frequency, when musicians usually don't? As we will see, the physics of sound waves - and especially frequency - affects the most basic aspects of music, including pitch, tuning, consonance and dissonance, harmony, and timbre.

Standing Waves and Musical Instruments
For middle school and up, an explanation of how standing waves in musical instruments produce sounds with particular pitches and timbres.

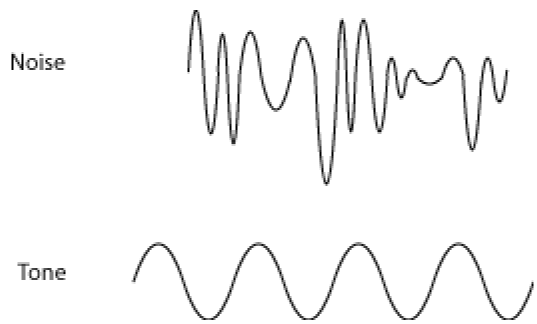
What is a Standing Wave?

Musical tones are produced by musical instruments, or by the voice, which, from a physics perspective, is a very complex wind instrument. So the physics of music is the physics of the kinds of sounds these instruments can make. What kinds of sounds are these? They are tones caused by standing waves produced in or on the instrument. So the properties of these standing waves, which are always produced in very specific groups, or series, have far-reaching effects on music theory.

Most sound waves, including the musical sounds that actually reach our ears, are not standing waves. Normally, when something makes a wave, the wave travels outward, gradually spreading out and losing strength, like the waves moving away from a pebble dropped into a pond.

But when the wave encounters something, it can bounce (reflection) or be bent (refraction). In fact, you can "trap" waves by making them bounce back and forth between two or more surfaces. Musical instruments take advantage of this; they produce pitches by trapping sound waves.

Why are trapped waves useful for music? Any bunch of sound waves will produce some sort of noise. But to be a **tone** - a sound with a particular pitch - a group of sound waves has to be very regular, all exactly the same distance apart. That's why we can talk about the frequency and wavelength of tones.



A noise is a jumble of sound waves. A tone is a very regular set of waves, all the same size and same distance apart.

So how can you produce a tone? Let's say you have a sound wave trap (for now, don't worry about what it looks like), and you keep sending more sound waves into it. Picture a lot of pebbles being dropped into a very small pool. As the waves start reflecting off the edges of the pond, they interfere with the new waves, making a jumble of waves that partly cancel each other out and mostly just roils the pond - noise.

But what if you could arrange the waves so that reflecting waves, instead of cancelling out the new waves, would reinforce them? The high parts of the reflected waves would meet the high parts of the oncoming waves and make them even higher. The low parts of the reflected waves would meet the low parts of the oncoming waves and make them even lower. Instead of a roiled mess of waves cancelling each other out, you would have a pond of perfectly ordered waves, with high points and low points appearing regularly at the same spots again and again. To help you imagine this, here are animations of a single wave reflecting back and forth and standing waves.

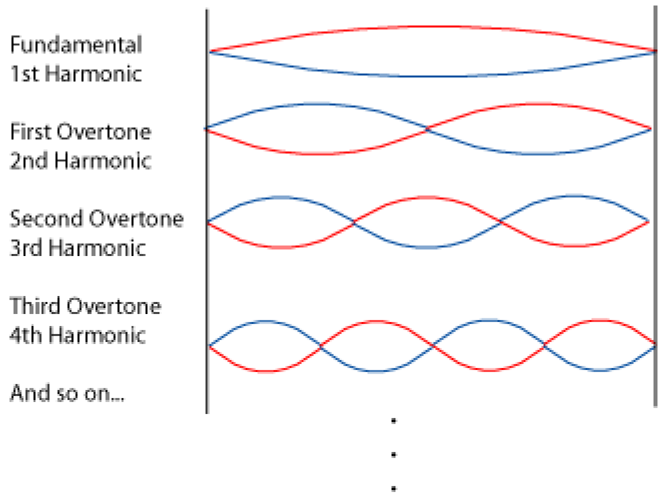
This sort of orderliness is actually hard to get from water waves, but relatively easy to get in sound waves, so that several completely different types of sound wave "containers" have been developed into musical

instruments. The two most common - strings and hollow tubes - will be discussed below, but first let's finish discussing what makes a good standing wave container, and how this affects music theory.

In order to get the necessary constant reinforcement, the container has to be the perfect size (length) for a certain wavelength, so that waves bouncing back or being produced at each end reinforce each other, instead of interfering with each other and cancelling each other out. And it really helps to keep the container very narrow, so that you don't have to worry about waves bouncing off the sides and complicating things. So you have a bunch of regularly-spaced waves that are trapped, bouncing back and forth in a container that fits their wavelength perfectly. If you could watch these waves, it would not even look as if they are traveling back and forth. Instead, waves would seem to be appearing and disappearing regularly at exactly the same spots, so these trapped waves are called **standing waves**.

Note: Although standing waves are harder to get in water, the phenomenon does apparently happen very rarely in lakes, resulting in freak disasters. You can sometimes get the same effect by pushing a tub of water back and forth, but this is a messy experiment; you'll know you are getting a standing wave when the water suddenly starts sloshing much higher - right out of the tub!

For any narrow "container" of a particular length, there are plenty of possible standing waves that don't fit. But there are also many standing waves that do fit. The longest wave that fits it is called the **fundamental**. It is also called the **first harmonic**. The next longest wave that fits is the **second harmonic**, or the **first overtone**. The next longest wave is the **third harmonic**, or **second overtone**, and so on.
Standing Wave Harmonics



There is a whole set of standing waves, called **harmonics**, that will fit into any "container" of a specific length. This set of waves is called a **harmonic series**.

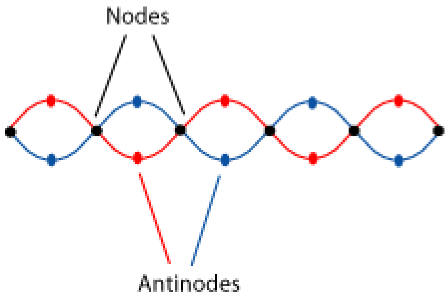
Notice that it doesn't matter what the length of the fundamental is; the waves in the second harmonic must be half the length of the first harmonic; that's the only way they'll both "fit". The waves of the third harmonic must be a third the length of the first harmonic, and so on. This has a direct effect on the frequency and pitch of harmonics, and so it affects the basics of music tremendously. To find out more about these subjects, please see Frequency, Wavelength, and Pitch, Harmonic Series, or Musical Intervals, Frequency, and Ratio.

Standing Waves on Strings

You may have noticed an interesting thing in the animation of standing waves: there are spots where the "water" goes up and down a great deal, and other spots where the "water level" doesn't seem to move at all. All standing waves have places, called **nodes**, where there is no wave motion, and **antinodes**, where the wave is largest. It is the placement of the nodes

that determines which wavelengths "fit" into a musical instrument "container".

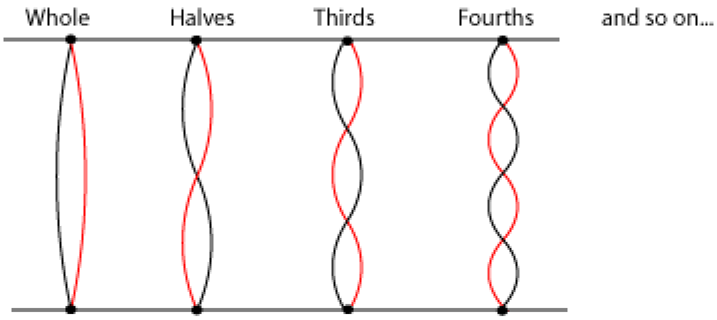
Nodes and Antinodes

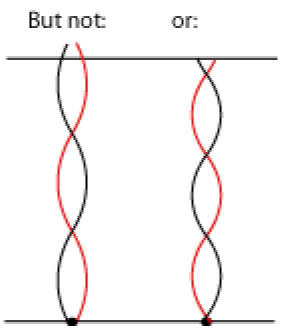


As a standing wave waves back and forth (from the red to the blue position), there are some spots called **nodes** that do not move at all; basically there is no change, no waving up-and-down (or back-and-forth), at these spots. The spots at the biggest part of the wave - where there is the most change during each wave - are called **antinodes**.

One "container" that works very well to produce standing waves is a thin, very taut string that is held tightly in place at both ends. Since the string is taut, it vibrates quickly, producing sound waves, if you pluck it, or rub it with a bow. Since it is held tightly at both ends, that means there has to be a node at each end of the string. Instruments that produce sound using strings are called chordophones, or simply strings.

Standing Waves on a String





A string that's held very tightly at both ends can only vibrate at very particular wavelengths. The whole string can vibrate back and forth. It can vibrate in halves, with a node at the middle of the string as well as each end, or in thirds, fourths, and so on. But any wavelength that doesn't have a node at each end of the string, can't make a standing wave on the string. To get any of those other wavelengths, you need to change the length of the vibrating string. That is what happens when the player holds the string down with a finger, changing the vibrating length of the string and changing where the nodes are.

The fundamental wave is the one that gives a string its pitch. But the string is making all those other possible vibrations, too, all at the same time, so

that the actual vibration of the string is pretty complex. The other vibrations (the ones that basically divide the string into halves, thirds and so on) produce a whole series of **harmonics**. We don't hear the harmonics as separate notes, but we do hear them. They are what gives the string its rich, musical, string-like sound - its timbre. (The sound of a single frequency alone is a much more mechanical, uninteresting, and unmusical sound.) To find out more about harmonics and how they affect a musical sound, see Harmonic Series.

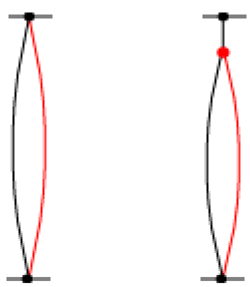
Exercise:

Problem:
When the string player puts a finger down tightly on the string,

1. How has the part of the string that vibrates changed?
2. How does this change the sound waves that the string makes?
3. How does this change the sound that is heard?

Solution:

1. The part of the string that can vibrate is shorter. The finger becomes the new "end" of the string.
2. The new sound wave is shorter, so its frequency is higher.
3. It sounds higher; it has a higher pitch.



When a finger holds the string down tightly,

the finger becomes the new end of the vibrating part of the string. The vibrating part of the string is shorter, and the whole set of sound waves it makes is shorter.

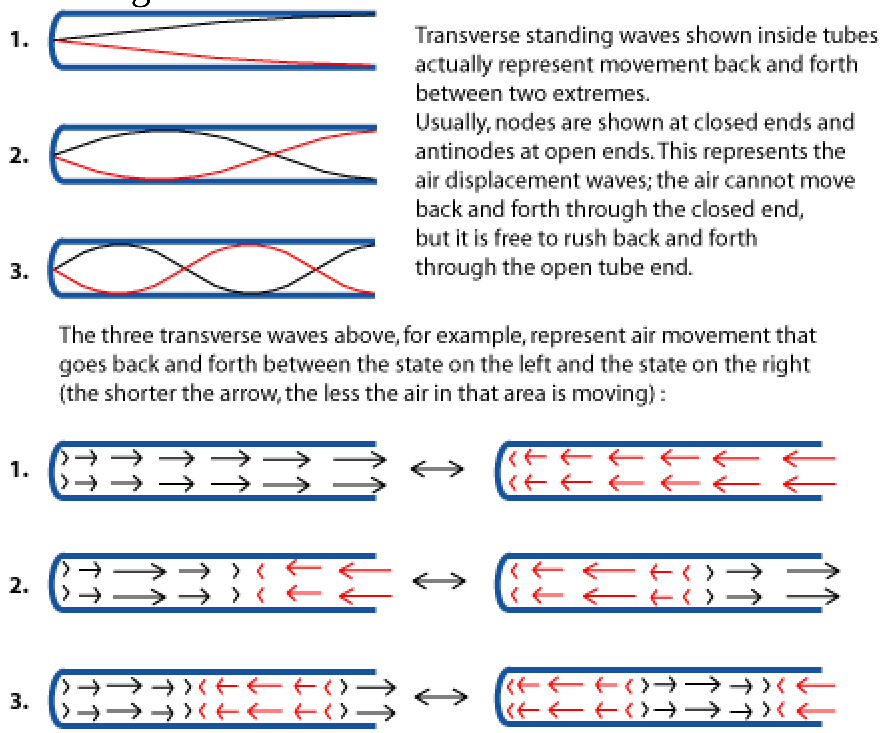
Standing Waves in Wind Instruments

The string disturbs the air molecules around it as it vibrates, producing sound waves in the air. But another great container for standing waves actually holds standing waves of air inside a long, narrow tube. This type of instrument is called an aerophone, and the most well-known of this type of instrument are often called wind instruments because, although the instrument itself does vibrate a little, most of the sound is produced by standing waves in the column of air inside the instrument.

If it is possible, have a reed player and a brass player demonstrate to you the sounds that their mouthpieces make without the instrument. This will be a much "noisier" sound, with lots of extra frequencies in it that don't sound very musical. But, when you put the mouthpiece on an instrument shaped like a tube, only some of the sounds the mouthpiece makes are the right

length for the tube. Because of feedback from the instrument, the only sound waves that the mouthpiece can produce now are the ones that are just the right length to become **standing waves** in the instrument, and the "noise" is refined into a musical tone.

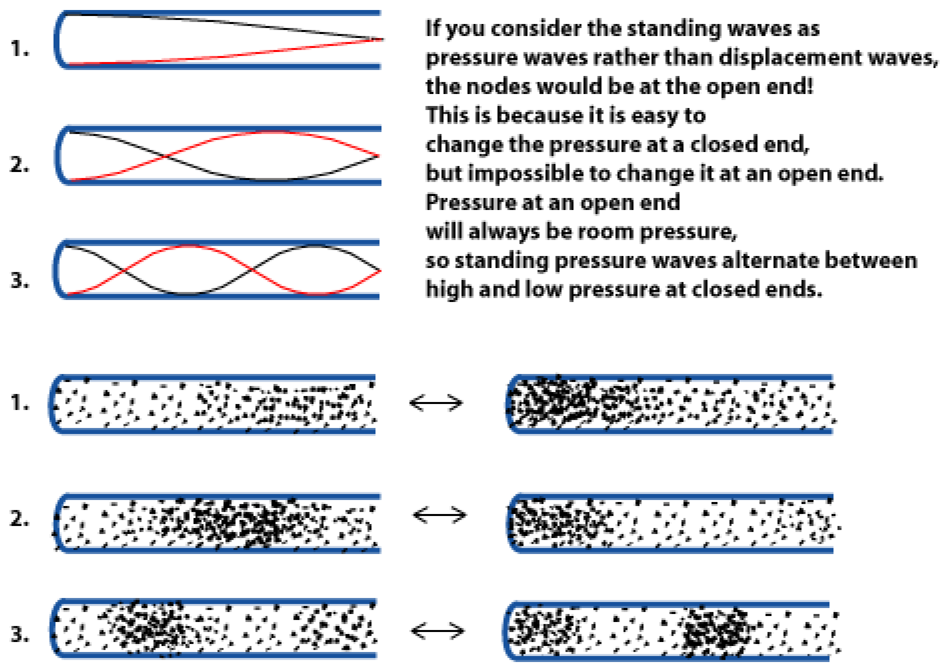
Standing Waves in Wind Instruments



Standing Waves in a wind instrument are usually shown as displacement waves, with nodes at closed ends where the air cannot move back-and-forth.

The standing waves in a wind instrument are a little different from a vibrating string. The wave on a string is a **transverse wave**, moving the string back and forth, rather than moving up and down along the string. But the wave inside a tube, since it is a sound wave already, is a **longitudinal wave**; the waves do not go from side to side in the tube. Instead, they form along the length of the tube.

Longitudinal Waves in Pipes



The standing waves in the tubes are actually longitudinal sound waves. Here the displacement standing waves in [link] are shown instead as longitudinal air pressure waves. Each wave would be oscillating back and forth between the state on the right and the one on the left. See Standing Waves in Wind Instruments for more explanation.

The harmonics of wind instruments are also a little more complicated, since there are two basic shapes (cylindrical and conical) that are useful for wind instruments, and they have different properties. The standing-wave tube of a wind instrument also may be open at both ends, or it may be closed at one end (for a mouthpiece, for example), and this also affects the instrument. Please see Standing Waves in Wind Instruments if you want more information on that subject. For the purposes of understanding music theory, however, the important thing about standing waves in winds is this: the harmonic series they produce is essentially the same as the harmonic series on a string. In other words, the second harmonic is still half the length of the fundamental, the third harmonic is one third the length, and so on. (Actually, for reasons explained in Standing Waves in Wind

Instruments, some harmonics are "missing" in some wind instruments, but this mainly affects the timbre and some aspects of playing the instrument. It does not affect the basic relationships in the harmonic series.)

Standing Waves in Other Objects

So far we have looked at two of the four main groups of musical instruments: chordophones and aerophones. That leaves membranophones and idiophones. **Membranophones** are instruments in which the sound is produced by making a membrane vibrate; drums are the most familiar example. Most drums do not produce tones; they produce rhythmic "noise" (bursts of irregular waves). Some drums do have pitch, due to complex-patterned standing waves on the membrane that are reinforced in the space inside the drum. This works a little bit like the waves in tubes, above, but the waves produced on membranes, though very interesting, are too complex to be discussed here.

Idiophones are instruments in which the body of the instrument itself, or a part of it, produces the original vibration. Some of these instruments (cymbals, for example) produce simple noise-like sounds when struck. But in some, the shape of the instrument - usually a tube, block, circle, or bell shape - allows the instrument to ring with a standing-wave vibration when you strike it. The standing waves in these carefully-shaped-and-sized idiophones - for example, the blocks on a xylophone - produce pitched tones, but again, the patterns of standing waves in these instruments are a little too complicated for this discussion. If a percussion instrument does produce pitched sounds, however, the reason, again, is that it is mainly producing harmonic-series overtones.

Note: Although percussion specializes in "noise"-type sounds, even instruments like snare drums follow the basic physics rule of "bigger instrument makes longer wavelengths and lower sounds". If you can, listen to a percussion player or section that is using snare drums, cymbals, or other percussion of the same type but different sizes. Can you hear the

difference that size makes, as opposed to differences in timbre produced by different types of drums?

Exercise:

Problem:

Some idiophones, like gongs, ring at many different pitches when they are struck. Like most drums, they don't have a particular pitch, but make more of a "noise"-type sound. Other idiophones, though, like xylophones, are designed to ring at more particular frequencies. Can you think of some other percussion instruments that get particular pitches? (Some can get enough different pitches to play a tune.)

Solution:

There are many, but here are some of the most familiar:

- Chimes
- All xylophone-type instruments, such as marimba, vibraphone, and glockenspiel
- Handbells and other tuned bells
- Steel pan drums

Harmonic Series
The harmonic series is the key to understanding not only harmonics, but also timbre and the basic functioning of many musical instruments.

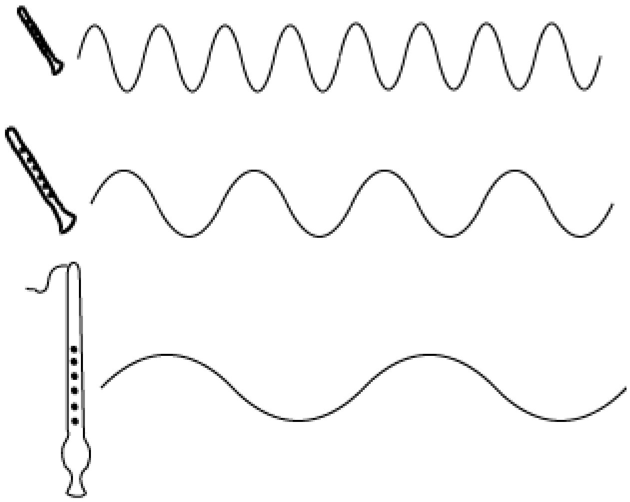
Introduction

Have you ever wondered how a trumpet plays so many different notes with only three valves, or how a bugle plays different notes with no valves at all? Have you ever wondered why an oboe and a flute sound so different, even when they're playing the same note? What is a string player doing when she plays "harmonics"? Why do some notes sound good together while other notes seem to clash with each other? The answers to all of these questions will become clear with an understanding of the harmonic series.

Physics, Harmonics and Color

Most musical notes are sounds that have a particular pitch. The pitch depends on the main frequency of the sound; the higher the frequency, and shorter the wavelength, of the sound waves, the higher the pitch is. But musical sounds don't have just one frequency. Sounds that have only one frequency are not very interesting or pretty. They have no more musical color than the beeping of a watch alarm. On the other hand, sounds that have too many frequencies, like the sound of glass breaking or of ocean waves crashing on a beach, may be interesting and even pleasant. But they don't have a particular pitch, so they usually aren't considered musical notes.

Frequency and Pitch



The higher the frequency, the higher the note sounds.

When someone plays or sings a note, only a very particular set of frequencies is heard. Imagine that each note that comes out of the instrument is a smooth mixture of many different pitches. These different pitches are called **harmonics**, and they are blended together so well that you do not hear them as separate notes at all. Instead, the harmonics give the note its color.

What is the color of a sound? Say an oboe plays a middle C. Then a flute plays the same note at the same loudness as the oboe. It is still easy to tell the two notes apart, because an oboe sounds different from a flute. This difference in the sounds is the **color**, or **timbre** (pronounced "TAM-ber") of the notes. Like a color you see, the color of a sound can be bright and bold or deep and rich. It can be heavy, light, murky, thin, smooth, or transparently clear. Some other words that musicians use to describe the timbre of a sound are: reedy, brassy, piercing, mellow, thin, hollow, focussed, breathy (pronounced BRETH-ee) or full. Listen to recordings of a violin and a viola. Although these instruments are quite similar, the viola has a noticeably "deeper" and the violin a noticeably "brighter" sound that is not simply a matter of the violin playing higher notes. Now listen to the same phrase played by an electric guitar, an acoustic guitar with twelve

steel strings and an acoustic guitar with six nylon strings. The words musicians use to describe timbre are somewhat subjective, but most musicians would agree with the statement that, compared with each other, the first sound is mellow, the second bright, and the third rich.

Exercise:

Problem:

Listen to recordings of different instruments playing alone or playing very prominently above a group. Some suggestions: an unaccompanied violin or cello sonata, a flute, oboe, trumpet, or horn concerto, native American flute music, classical guitar, bagpipes, steel pan drums, panpipes, or organ. For each instrument, what "color" words would you use to describe the timbre of each instrument? Use as many words as you can that seem appropriate, and try to think of some that aren't listed above. Do any of the instruments actually make you think of specific shades of color, like fire-engine red or sky blue?

Solution:

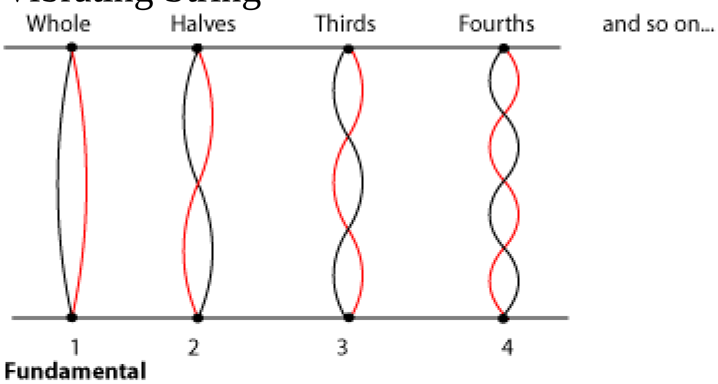
Although trained musicians will generally agree that a particular sound is reedy, thin, or full, there are no hard-and-fast right-and-wrong answers to this exercise.

Where do the harmonics, and the timbre, come from? When a string vibrates, the main pitch you hear is from the vibration of the whole string back and forth. That is the **fundamental**, or first harmonic. But the string also vibrates in halves, in thirds, fourths, and so on. Each of these fractions also produces a harmonic. The string vibrating in halves produces the second harmonic; vibrating in thirds produces the third harmonic, and so on.

Note: This method of naming and numbering harmonics is the most straightforward and least confusing, but there are other ways of naming and numbering harmonics, and this can cause confusion. Some musicians do not consider the fundamental to be a harmonic; it is just the

fundamental. In that case, the string halves will give the first harmonic, the string thirds will give the second harmonic and so on. When the fundamental is included in calculations, it is called the first **partial**, and the rest of the harmonics are the second, third, fourth partials and so on. Also, some musicians use the term **overtones** as a synonym for harmonics. For others, however, an overtone is any frequency (not necessarily a harmonic) that can be heard resonating with the fundamental. The sound of a gong or cymbals will include overtones that aren't harmonics; that's why the gong's sound doesn't seem to have as definite a pitch as the vibrating string does. If you are uncertain what someone means by the second harmonic or by the term overtones, ask for clarification.

Vibrating String



The fundamental pitch is produced by the whole string vibrating back and forth. But the string is also vibrating in halves, thirds, quarters, fifths, and so on, producing **harmonics**. All of these vibrations happen at the same time, producing a rich, complex, interesting sound.

A column of air vibrating inside a tube is different from a vibrating string, but the column of air can also vibrate in halves, thirds, fourths, and so on, of

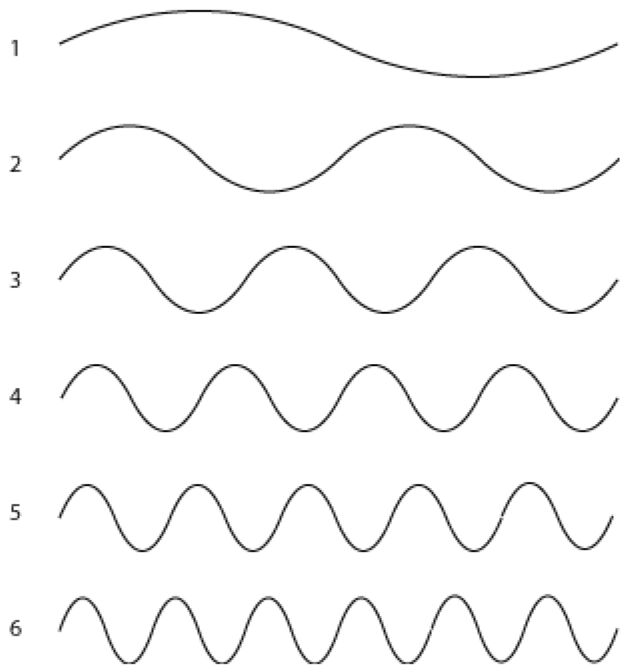
the fundamental, so the harmonic series will be the same. So why do different instruments have different timbres? The difference is the relative loudness of all the different harmonics compared to each other. When a clarinet plays a note, perhaps the odd-numbered harmonics are strongest; when a French horn plays the same notes, perhaps the fifth and tenth harmonics are the strongest. This is what you hear that allows you to recognize that it is a clarinet or horn that is playing.

Note: You will find some more extensive information on instruments and harmonics in Standing Waves and Musical Instruments and Standing Waves and Wind Instruments.

The Harmonic Series

A harmonic series can have any note as its fundamental, so there are many different harmonic series. But the relationship between the frequencies of a harmonic series is always the same. The second harmonic always has exactly half the wavelength (and twice the frequency) of the fundamental; the third harmonic always has exactly a third of the wavelength (and so three times the frequency) of the fundamental, and so on. For more discussion of wavelengths and frequencies, see Frequency, Wavelength, and Pitch.

Harmonic Series Wavelengths and Frequencies



The second harmonic has half the wavelength and twice the frequency of the first. The third harmonic has a third the wavelength and three times the frequency of the first. The fourth harmonic has a quarter the wavelength and four times the frequency of the first, and so on. Notice that the fourth harmonic is also twice the frequency of the second harmonic, and the sixth harmonic is also twice the frequency of the third harmonic.

Say someone plays a note, a middle C. Now someone else plays the note that is twice the frequency of the middle C. Since this second note was already a harmonic of the first note, the sound waves of the two notes reinforce each other and sound good together. If the second person played instead the note that was just a litle bit more than twice the frequency of the

first note, the harmonic series of the two notes would not fit together at all, and the two notes would not sound as good together. There are many combinations of notes that share some harmonics and make a pleasant sound together. They are considered consonant. Other combinations share fewer or no harmonics and are considered dissonant or, when they really clash, simply "out of tune" with each other. The scales and chords of most of the world's musics are based on these physical facts.

Note: In real music, consonance and dissonance also depend on the standard practices of a musical tradition, especially its harmony practices, but these are also often related to the harmonic series.

For example, a note that is twice the frequency of another note is one octave higher than the first note. So in the figure above, the second harmonic is one octave higher than the first; the fourth harmonic is one octave higher than the second; and the sixth harmonic is one octave higher than the third.

Exercise:

Problem:

1. Which harmonic will be one octave higher than the fourth harmonic?
2. Predict the next four sets of octaves in a harmonic series.
3. What is the pattern that predicts which notes of a harmonic series will be one octave apart?
4. Notes one octave apart are given the same name. So if the first harmonic is a "A", the second and fourth will also be A's. Name three other harmonics that will also be A's.

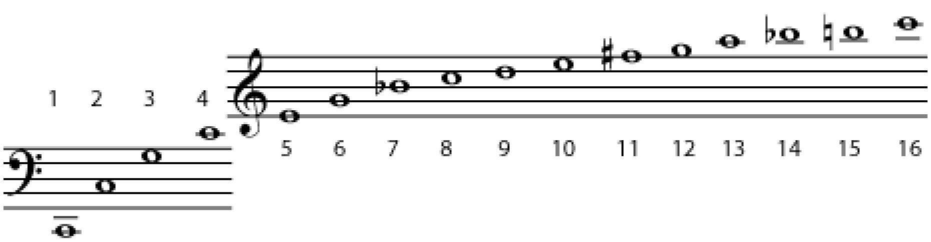
Solution:

1. The eighth harmonic
2. The fifth and tenth harmonics; the sixth and twelfth harmonics; the seventh and fourteenth harmonics; and the eighth and

sixteenth harmonics
3. The note that is one octave higher than a harmonic is also a harmonic, and its number in the harmonic series is twice (2 X) the number of the first note.
4. The eighth, sixteenth, and thirty-second harmonics will also be A's.

A mathematical way to say this is "if two notes are an octave apart, the ratio of their frequencies is two to one (2:1)". Although the notes themselves can be any frequency, the 2:1 ratio is the same for all octaves. And all the other intervals that musicians talk about can also be described as being particular ratios of frequencies.

A Harmonic Series Written as Notes



Take the third harmonic, for example. Its frequency is three times the first harmonic (ratio 3:1). Remember, the frequency of the second harmonic is two times that of the first harmonic. So the ratio of the frequencies of the second to the third harmonics is 2:3. From the harmonic series shown above, you can see that the interval between these two notes is a perfect fifth. The ratio of the frequencies of all perfect fifths is 2:3.

Exercise:

Problem:

1. The interval between the fourth and sixth harmonics (frequency ratio 4:6) is also a fifth. Can you explain this?
2. What other harmonics have an interval of a fifth?
3. Which harmonics have an interval of a fourth?
4. What is the frequency ratio for the interval of a fourth?

Solution:

1. The ratio 4:6 reduced to lowest terms is 2:3. (If you are more comfortable with fractions than with ratios, think of all the ratios as fractions instead. 2:3 is just two-thirds, and 4:6 is four-sixths. Four-sixths reduces to two-thirds.)
2. Six and nine (6:9 also reduces to 2:3); eight and twelve; ten and fifteen; and any other combination that can be reduced to 2:3 (12:18, 14:21 and so on).
3. Harmonics three and four; six and eight; nine and twelve; twelve and sixteen; and so on.
4. 3:4

Note: If you have been looking at the harmonic series above closely, you may have noticed that some notes that are written to give the same interval have different frequency ratios. For example, the interval between the seventh and eighth harmonics is a major second, but so are the intervals between 8 and 9, between 9 and 10, and between 10 and 11. But 7:8, 8:9, 9:10, and 10:11, although they are pretty close, are not exactly the same. In fact, modern Western music uses the equal temperament tuning system, which divides the octave into twelve notes that are spaced equally far apart. The positive aspect of equal temperament (and the reason it is used) is that an instrument will be equally in tune in all keys. The negative aspect is that it means that all intervals except for octaves are slightly out of tune with regard to the actual harmonic series. For more about equal temperament, see Tuning Systems. Interestingly, musicians have a tendency to revert to true harmonics when they can (in other words, when it is easy to fine-tune each note). For example, an a capella choral group or a brass ensemble, may find themselves singing or playing perfect fourths and fifths, "contracted" major thirds and "expanded" minor thirds.

Brass Instruments

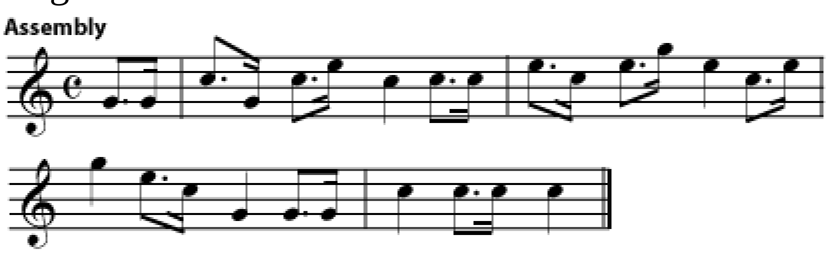
The harmonic series is particularly important for brass instruments. A pianist or xylophone player only gets one note from each key. A string player who wants a different note from a string holds the string tightly in a different place. This basically makes a vibrating string of a new length, with a new fundamental.

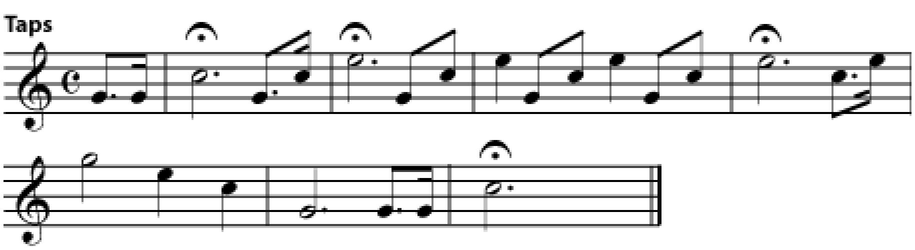
But a brass player, without changing the length of the instrument, gets different notes by actually playing the harmonics of the instrument. Woodwinds also do this, although not as much. Most woodwinds can get two different octaves with essentially the same fingering; the lower octave is the fundamental of the column of air inside the instrument at that fingering. The upper octave is the first harmonic.

But it is the brass instruments that excel in getting different notes from the same length of tubing. The sound of a brass instruments starts with vibrations of the player's lips. By vibrating the lips at different speeds, the player can cause a harmonic of the air column to sound instead of the fundamental.

So a bugle player can play any note in the harmonic series of the instrument that falls within the player's range. Compare these well-known bugle calls to the harmonic series above.

Bugle Calls





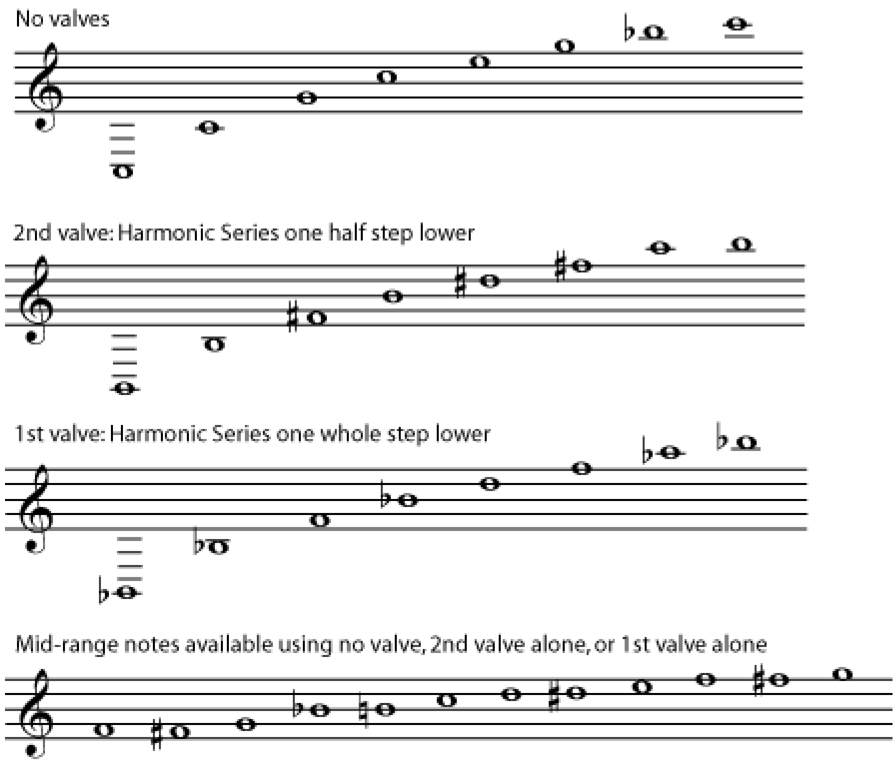
Although limited by the fact that it can only play

one harmonic series, the bugle can still play many well-known tunes.

For centuries, all brass instruments were valveless. A brass instrument could play only the notes of one harmonic series. The upper octaves of the series, where the notes are close together, could be difficult or impossible to play, and some of the harmonics sound quite out of tune to ears that expect equal temperament. The solution to these problems, once brass valves were perfected, was to add a few valves to the instrument. Three is usually enough. Each valve opens an extra length of tube, making the instrument a little longer, and making available a whole new harmonic series. Usually one valve gives the harmonic series one half step lower than the valveless intrument, another one whole step lower, and another one and a half steps lower. The valves can be used at the same time, too, making even more harmonic series. So a valved brass instrument can find, in the comfortable middle of its range (its **middle register**), a valve combination that will give a reasonably in-tune version for every note of the chromatic scale. (For more on the history of valved brass, see History of the French Horn. For more on how and why harmonics are produced in wind instruments, please see Standing Waves and Wind Instruments)

> **Note:**Trombones use a slide instead of valves to make their instrument longer. But the basic principle is still the same. At each slide "position", the instrument gets a new harmonic series. The notes in between the positions aren't part of the chromatic scale, so they are usually only used for special effects like **glissandos** (sliding notes).

Overlapping Harmonic Series in Brass Instruments



These harmonic series are for a brass instrument that has a "C" fundamental when no valves are being used - for example, a C trumpet. Remember, there is an entire harmonic series for every fundamental, and any note can be a fundamental. You just have to find the brass tube with the right length. So a trumpet or tuba can get one harmonic series using no valves, another one a half step lower using one valve, another one a whole step lower using another valve, and so on. By the time all the combinations of valves are used, there is some way to get an in-tune version of every note they need.

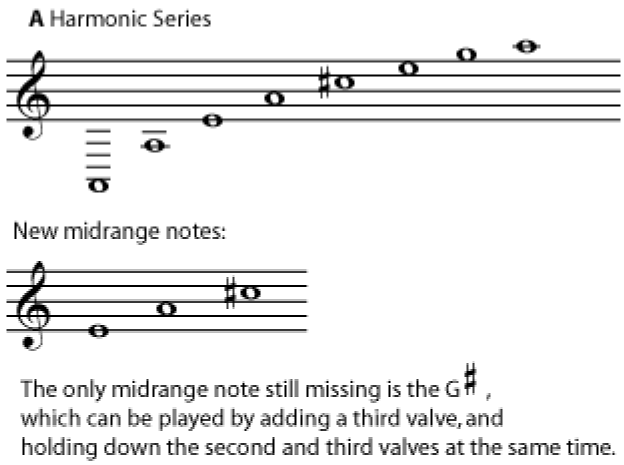
Exercise:

Problem:

Write the harmonic series for the instrument above when both the first and second valves are open. (You can use this PDF file if you need staff paper.) What new notes are added in the instrument's middle range? Are any notes still missing?

Solution:

Opening both first and second valves gives the harmonic series one-and-a-half steps lower than "no valves".



Note: The French horn has a reputation for being a "difficult" instrument to play. This is also because of the harmonic series. Most brass instruments play in the first few octaves of the harmonic series, where the notes are farther apart and it takes a pretty big difference in the mouth and lips (the embouchure, pronounced AHM-buh-sher) to get a different note. The range of the French horn is higher in the harmonic series, where the notes are closer together. So very small differences in the mouth and lips can mean the wrong harmonic comes out.

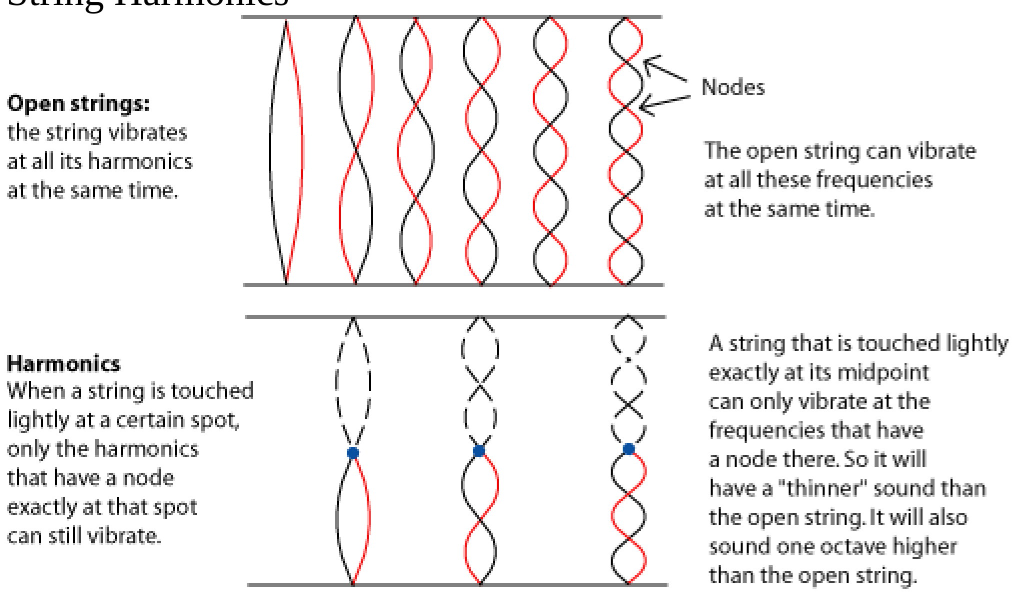
Playing Harmonics on Strings

String players also use harmonics, although not as much as brass players. Harmonics on strings have a very different timbre from ordinary string sounds. They give a quieter, thinner, more bell-like tone, and are usually used as a kind of ear-catching-special-effect.

Normally when a string player puts a finger on a string, he holds it down tight. This basically forms a (temporarily) shorter vibrating string, which then produces an entire harmonic series, with a shorter (higher) fundamental.

In order to play a harmonic, he touches the string very, very lightly instead. So the length of the string does not change. Instead, the light touch interferes with all of the vibrations that don't have a node at that spot. (A **node** is a place in the wave where the string does not move back-and-forth. For example, the ends of the string are both nodes, since they are held in place.)

String Harmonics

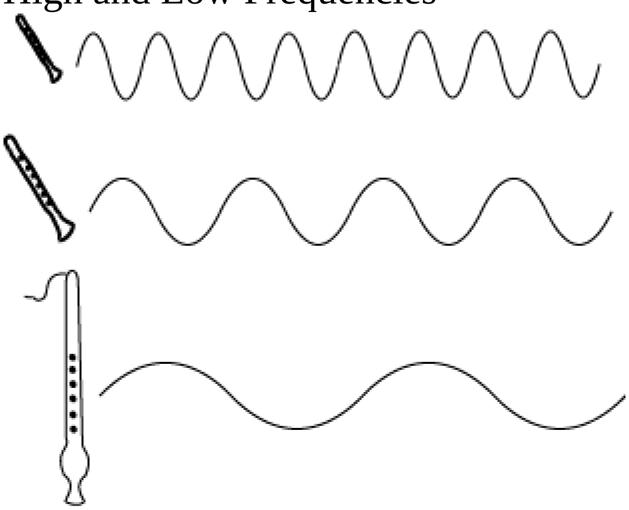


The thinner, quieter sound of "playing harmonics" is caused by the fact that much of the harmonic series is missing from the sound, which will of course be heard in the timbre. Lightly touching the string in most spots will result in no sound at all. It only works at the precise spots that will leave some of the main harmonics (the longer, louder, lower-numbered ones) free to vibrate.

Octaves and the Major-Minor Tonal System
Introduces the relationship between frequency, octaves, major, minor, and chromatic scales, and tonal music.

Where Octaves Come From

Musical notes, like all sounds, are made of sound waves. The sound waves that make musical notes are very evenly-spaced waves, and the qualities of these regular waves - for example how big they are or how far apart they are - affect the sound of the note. A note can be high or low, depending on how often (how frequently) one of its waves arrives at your ear. When scientists and engineers talk about how high or low a sound is, they talk about its frequency. The higher the **frequency** of a note, the higher it sounds. They can measure the frequency of notes, and like most measurements, these will be numbers, like "440 vibrations per second."
High and Low Frequencies



A sound that has a shorter wavelength has a higher frequency and a higher pitch.

But people have been making music and talking about music since long before we knew that sounds were waves with frequencies. So when musicians talk about how high or low a note sounds, they usually don't talk

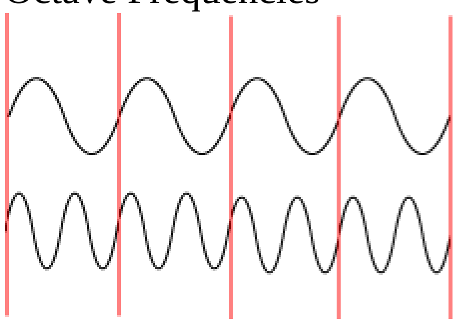
about frequency; they talk about the note's pitch. And instead of numbers, they give the notes names, like "C". (For example, musicians call the note with frequency "440 vibrations per second" an "A".)

But to see where octaves come from, let's talk about frequencies a little more. Imagine a few men are singing a song together. Nobody is singing harmony; they are all singing the same pitch - the same frequency - for each note.

Now some women join in the song. They can't sing where the men are singing; that's too low for their voices. Instead they sing notes that are exactly double the frequency that the men are singing. That means their note has exactly two waves for each one wave that the men's note has. These two frequencies fit so well together that it sounds like the women are singing the same notes as the men, in the same key. They are just singing them one octave higher. **Any note that is twice the frequency of another note is one octave higher.**

Notes that are one octave apart are so closely related to each other that musicians give them the same name. A note that is an octave higher or lower than a note named "C natural" will also be named "C natural". A note that is one (or more) octaves higher or lower than an "F sharp" will also be an "F sharp". (For more discussion of how notes are related because of their frequencies, see The Harmonic Series, Standing Waves and Musical Instruments, and Standing Waves and Wind Instruments.)

Octave Frequencies



When two notes are one octave apart, one has a frequency exactly two times

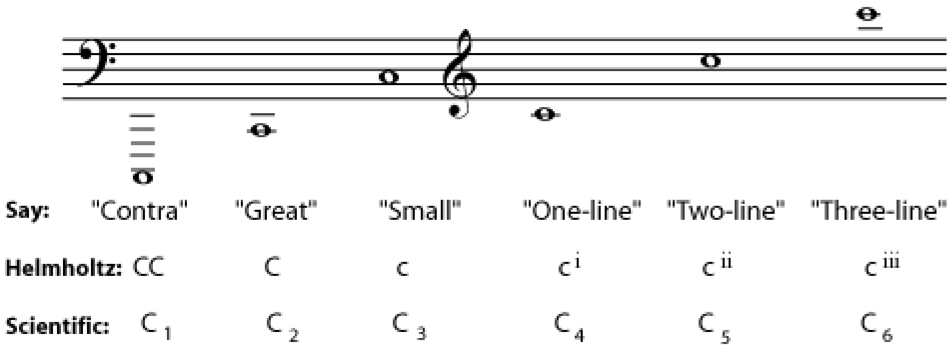
higher than the other - it has twice as many waves. These waves fit together so well, in the instrument, and in the air, and in your ears, that they sound almost like different versions of the same note.

Naming Octaves

The notes in different octaves are so closely related that when musicians talk about a note, a "G" for example, it often doesn't matter which G they are talking about. We can talk about the "F sharp" in a G major scale without mentioning which octave the scale or the F sharp are in, because the scale is the same in every octave. Because of this, many discussions of music theory don't bother naming octaves. Informally, musicians often speak of "the B on the staff" or the "A above the staff", if it's clear which staff they're talking about.

But there are also two formal systems for naming the notes in a particular octave. Many musicians use **Helmholtz** notation. Others prefer **scientific pitch notation**, which simply labels the octaves with numbers, starting with C1 for the lowest C on a full-sized keyboard. Figure 3 shows the names of the octaves most commonly used in music.

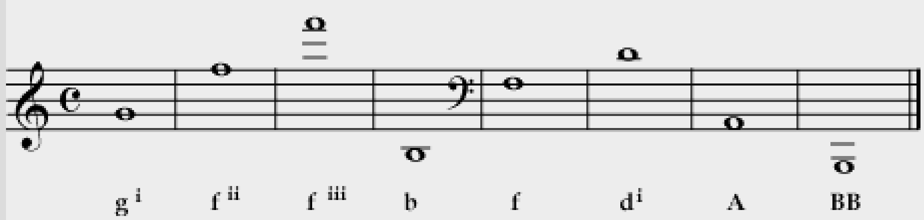
Naming Octaves



The octaves are named from one C to the next higher C. For example, all the notes in between "one line c" and "two line c" are "one line" notes.

The octave below contra can be labelled CCC or Co; higher octaves can be labelled with higher numbers or more lines. Octaves are named from one C to the next higher C. For example, all the notes between "great C" and "small C" are "great". **One-line c is also often called "middle C". No other notes are called "middle", only the C.**

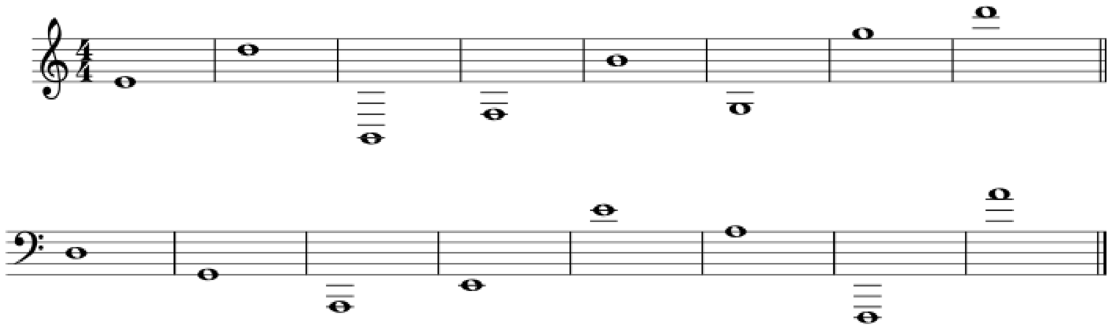
Example:
Naming Notes within a Particular Octave



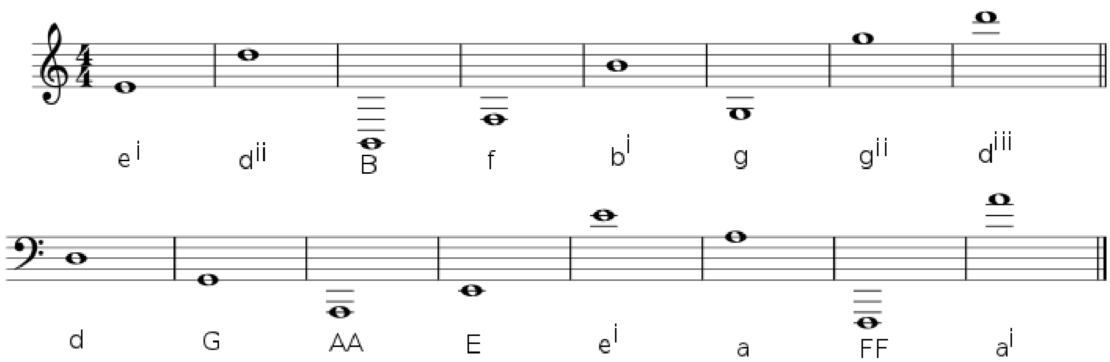
Each note is considered to be in the same octave as the C below it.

Exercise:

Problem: Give the correct octave name for each note.



Solution:



Dividing the Octave into Scales

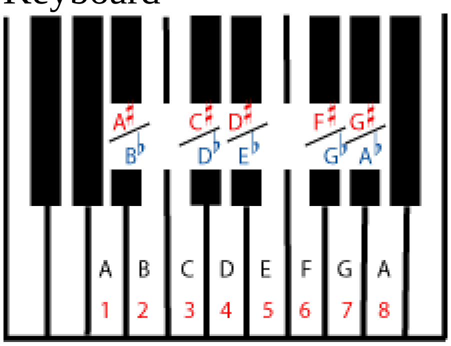
The word "octave" comes from a Latin root meaning "eight". It seems an odd name for a frequency that is two times, not eight times, higher. The octave was named by musicians who were more interested in how octaves are divided into scales, than in how their frequencies are related. Octaves aren't the only notes that sound good together. The people in different musical traditions have different ideas about what notes they think sound best together. In the Western musical tradition - which includes most

familiar music from Europe and the Americas - the octave is divided up into twelve equally spaced notes. If you play all twelve of these notes within one octave you are playing a chromatic scale. Other musical traditions - traditional Chinese music for example - have divided the octave differently and so they use different scales. (Please see Major Keys and Scales, Minor Keys and Scales, and Scales that aren't Major or Minor for more about this.)

You may be thinking "OK, that's twelve notes; that still has nothing to do with the number eight", but out of those twelve notes, only seven are used in any particular major or minor scale. Add the first note of the next octave, so that you have that a "complete"-sounding scale ("do-re-mi-fa-so-la-ti" and then "do" again), and you have the eight notes of the **octave**. These are the **diatonic** scales, and they are the basis of most Western music.

Now take a look at the piano keyboard. Only seven letter names are used to name notes: A, B, C, D, E, F, and G. The eighth note would, of course, be the next A, beginning the next octave. To name the other notes, the notes on the black piano keys, you have to use a sharp or flat sign.

Keyboard



The white keys are the natural notes. Black keys can only be named using sharps or flats. The pattern repeats at the eighth tone of a scale, the octave.

Whether it is a popular song, a classical symphony, or an old folk tune, most of the music that feels comfortable and familiar (to Western listeners) is based on either a major or minor scale. It is **tonal** music that mostly uses only seven of the notes within an octave: only one of the possible A's (A sharp, A natural, or A flat), one of the possible B's (B sharp, B natural, or B flat), and so on. The other notes in the chromatic scale are (usually) used sparingly to add interest or to (temporarily) change the key in the middle of the music. For more on the keys and scales that are the basis of tonal music, see Major Keys and Scales and Minor Keys and Scales.

Tuning Systems
An overview of music tuning systems

Introduction

The first thing musicians must do before they can play together is "tune". For musicians in the standard Western music tradition, this means agreeing on exactly what pitch (what frequency) is an "A", what is a "B flat" and so on. Other cultures not only have different note names and different scales, they may even have different notes - different pitches - based on a different tuning system. In fact, the modern Western tuning system, which is called **equal temperament**, replaced (relatively recently) other tuning systems that were once popular in Europe. All tuning systems are based on the physics of sound. But they all are also affected by the history of their music traditions, as well as by the tuning peculiarities of the instruments used in those traditions. Pythagorean, mean-tone, just intonation, well temperaments, equal temperament, and wide tuning.

To understand all of the discussion below, you must be comfortable with both the musical concept of interval and the physics concept of frequency. If you wish to follow the whole thing but are a little hazy on the relationship between pitch and frequency, the following may be helpful: Pitch; Acoustics for Music Theory; Harmonic Series I: Timbre and Octaves; and Octaves and the Major-Minor Tonal System. If you do not know what intervals are (for example, major thirds and perfect fourths), please see Interval and Harmonic Series II: Harmonics, Intervals and Instruments. If you need to review the mathematical concepts, please see Musical Intervals, Frequency, and Ratio and Powers, Roots, and Equal Temperament. Meanwhile, here is a reasonably nontechnical summary of the information below: Modern Western music uses the equal temperament tuning system. In this system, an octave (say, from C to C) is divided into twelve equally-spaced notes. "Equally-spaced" to a musician basically means that each of these notes is one half step from the next, and that all half steps sound like the same size pitch change. (To a scientist or engineer, "equally-spaced" means that the ratio of the frequencies of the two notes in any half step is always the same.) This tuning system is very convenient for some instruments, such as the piano, and also makes it very easy to change key

without retuning instruments. But a careful hearing of the music, or a look at the physics of the sound waves involved, reveals that equal-temperament pitches are not based on the harmonics physically produced by any musical sound. The "equal" ratios of its half steps are the twelfth root of two, rather than reflecting the simpler ratios produced by the sounds themselves, and the important intervals that build harmonies can sound slightly out of tune. This often leads to some "tweaking" of the tuning in real performances, away from equal temperament. It also leads many other music traditions to prefer tunings other than equal temperament, particularly tunings in which some of the important intervals are based on the pure, simple-ratio intervals of physics. In order to feature these favored intervals, a tuning tradition may do one or more of the following: use scales in which the notes are not equally spaced; avoid any notes or intervals which don't work with a particular tuning; change the tuning of some notes when the key or mode changes.

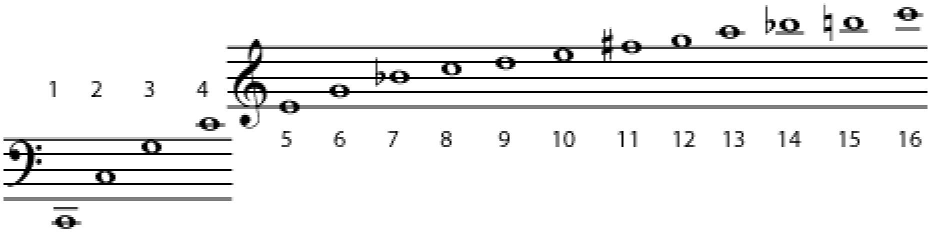
Tuning based on the Harmonic Series

Almost all music traditions recognize the octave. When note Y has a frequency that is twice the frequency of note Z, then note Y is one octave higher than note Z. A simple mathematical way to say this is that the ratio of the frequencies is 2:1. Two notes that are exactly one octave apart sound good together because their frequencies are related in such a simple way. If a note had a frequency, for example, that was 2.11 times the frequency of another note (instead of exactly 2 times), the two notes would not sound so good together. In fact, most people would find the effect very unpleasant and would say that the notes are not "in tune" with each other.

To find other notes that sound "in tune" with each other, we look for other sets of pitches that have a "simple" frequency relationship. These sets of pitches with closely related frequencies are often written in common notation as a harmonic series. The harmonic series is not just a useful idea constructed by music theory; it is often found in "real life", in the real-world physics of musical sounds. For example, a bugle can play only the notes of a specific harmonic series. And every musical note you hear is not a single pure frequency, but is actually a blend of the pitches of a particular harmonic series. The relative strengths of the harmonics are what gives the

note its timbre. (See Harmonic Series II: Harmonics, Intervals and Instruments; Standing Waves and Musical Instruments; and Standing Waves and Wind Instruments for more about how and why musical sounds are built from harmonic series.)

Harmonic Series on C



Here are the first sixteen pitches in a harmonic series that starts on a C natural. The series goes on indefinitely, with the pitches getting closer and closer together. A harmonic series can start on any note, so there are many harmonic series, but **every harmonic series has the same set of intervals and the same frequency ratios**.

What does it mean to say that two pitches have a "simple frequency relationship"? It doesn't mean that their frequencies are almost the same. Two notes whose frequencies are almost the same - say, the frequency of one is 1.005 times the other - sound bad together. Again, anyone who is accustomed to precise tuning would say they are "out of tune". Notes with a close relationship have frequencies that can be written as a ratio of two small whole numbers; the smaller the numbers, the more closely related the notes are. Two notes that are exactly the same pitch, for example, have a frequency ratio of 1:1, and octaves, as we have already seen, are 2:1. Notice that when two pitches are related in this simple-ratio way, it means that they can be considered part of the same harmonic series, and in fact the actual harmonic series of the two notes may also overlap and reinforce each other. The fact that the two notes are complementing and reinforcing each other in this way, rather than presenting the human ear with two completely

different harmonic series, may be a major reason why they sound consonant and "in tune".

Note: Nobody has yet proven a physical basis for why simple-ratio combinations sound pleasant to us. For a readable introduction to the subject, I suggest Robert Jourdain's *Music, the Brain, and Ecstasy*

Notice that the actual frequencies of the notes do not matter. What matters is how they compare to each other - basically, how many waves of one note go by for each wave of the other note. Although the actual frequencies of the notes will change for every harmonic series, the comparative distance between the notes, their interval, will be the same.

For more examples, look at the harmonic series in [link]. The number beneath a note tells you the relationship of that note's frequency to the frequency of the first note in the series - the **fundamental**. For example, the frequency of the note numbered 3 in [link] is three times the frequency of the fundamental, and the frequency of the note numbered fifteen is fifteen times the frequency of the fundamental. In the example, the fundamental is a C. That note's frequency times 2 gives you another C; times 2 again (4) gives another C; times 2 again gives another C (8), and so on. Now look at the G's in this series. The first one is number 3 in the series. 3 times 2 is 6, and number 6 in the series is also a G. So is number 12 (6 times 2). Check for yourself the other notes in the series that are an octave apart. You will find that the ratio for one octave is always 2:1, just as the ratio for a unison is always 1:1. Notes with this small-number ratio of 2:1 are so closely related that we give them the same name, and most tuning systems are based on this octave relationship.

The next closest relationship is the one based on the 3:2 ratio, the interval of the perfect fifth (for example, the C and G in the example harmonic series). The next lowest ratio, 4:3, gives the interval of a perfect fourth. Again, these pitches are so closely related and sound so good together that their intervals have been named "perfect". The perfect fifth figures prominently

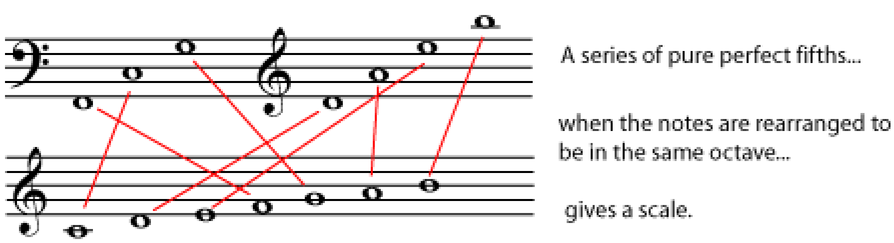
in many tuning systems. In Western music, all major and minor chords contain, or at least strongly imply, a perfect fifth. (See Triads and Naming Triads for more about the intervals in major and minor chords.)

Pythagorean Intonation

The Pythagorean system is so named because it was actually discussed by Pythagoras, the famous Greek mathematician and philosopher, who in the sixth century B.C. already recognized the simple arithmetical relationship involved in intervals of octaves, fifths, and fourths. He and his followers believed that numbers were the ruling principle of the universe, and that musical harmonies were a basic expression of the mathematical laws of the universe. Their model of the universe involved the "celestial spheres" creating a kind of harmony as they moved in circles dictated by the same arithmetical relationships as musical harmonies.

In the Pythagorean system, all tuning is based on the interval of the pure fifth. **Pure intervals** are the ones found in the harmonic series, with very simple frequency ratios. So a pure fifth will have a frequency ratio of exactly 3:2. Using a series of perfect fifths (and assuming perfect octaves, too, so that you are filling in every octave as you go), you can eventually fill in an entire chromatic scale.

Pythagorean Intonation



You can continue this series of perfect fifths to get the rest of the notes of a chromatic scale; the series would continue F sharp, C sharp, and so on.

The main weakness of the Pythagorean system is that a series of pure perfect fifths will never take you to a note that is a pure octave above the note you started on. To see why this is a problem, imagine beginning on a C. A series of perfect fifths would give: C, G, D, A, E, B, F sharp, C sharp, G sharp, D sharp, A sharp, E sharp, and B sharp. In equal temperament (which doesn't use pure fifths), that B sharp would be exactly the same pitch as the C seven octaves above where you started (so that the series can, in essence, be turned into a closed loop, the Circle of Fifths). Unfortunately, the B sharp that you arrive at after a series of pure fifths is a little higher than that C.

So in order to keep pure octaves, instruments that use Pythagorean tuning have to use eleven pure fifths and one smaller fifth. The smaller fifth has traditionally been called a **wolf** fifth because of its unpleasant sound. Keys that avoid the wolf fifth sound just fine on instruments that are tuned this way, but keys in which the wolf fifth is often heard become a problem. To avoid some of the harshness of the wolf intervals, some harpsichords and other keyboard instruments were built with split keys for D sharp/E flat and for G sharp/A flat. The front half of the key would play one note, and the back half the other (differently tuned) note.

Pythagorean tuning was widely used in medieval and Renaissance times. Major seconds and thirds are larger in Pythagorean intonation than in equal temperament, and minor seconds and thirds are smaller. Some people feel that using such intervals in medieval music is not only more authentic, but sounds better too, since the music was composed for this tuning system.

More modern Western music, on the other hand, does not sound pleasant using Pythagorean intonation. Although the fifths sound great, the thirds are simply too far away from the pure major and minor thirds of the harmonic series. In medieval music, the third was considered a dissonance and was used sparingly - and actually, when you're using Pythagorean tuning, it really is a dissonance - but most modern harmonies are built from thirds (see Triads). In fact, the common harmonic tradition that includes everything from Baroque counterpoint to modern rock is often called **triadic harmony**.

Some modern Non-Western music traditions, which have a very different approach to melody and harmony, still base their tuning on the perfect fifth. Wolf fifths and ugly thirds are not a problem in these traditions, which build each mode within the framework of the perfect fifth, retuning for different modes as necessary. To read a little about one such tradition, please see Indian Classical Music: Tuning and Ragas.

Mean-tone System

The mean-tone system, in order to have pleasant-sounding thirds, takes rather the opposite approach from the Pythagorean. It uses the pure major third. In this system, the whole tone (or whole step) is considered to be exactly half of the pure major third. This is the **mean**, or average, of the two tones, that gives the system its name. A semitone (or half step) is exactly half (another mean) of a whole tone.

These smaller intervals all work out well in mean-tone tuning, but the result is a fifth that is noticeably smaller than a pure fifth. And a series of pure thirds will also eventually not line up with pure octaves, so an instrument tuned this way will also have a problem with wolf intervals.

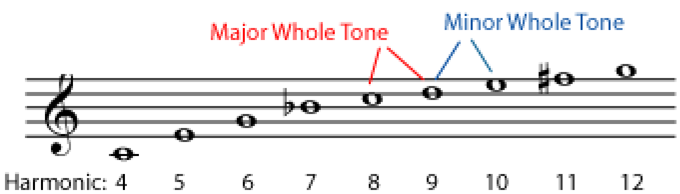
As mentioned above, Pythagorean tuning made sense in medieval times, when music was dominated by fifths. Once the concept of harmony in thirds took hold, thirds became the most important interval; simple perfect fifths were now heard as "austere" and, well, medieval-sounding. So mean-tone tuning was very popular in Europe in the 16th through 18th centuries.

But fifths can't be avoided entirely. A basic major or minor chord, for example, is built of two thirds, but it also has a perfect fifth between its outer two notes (see Triads). So even while mean-tone tuning was enjoying great popularity, some composers and musicians were searching for other solutions.

Just Intonation

In just intonation, the fifth and the third are both based on the pure, harmonic series interval. Because chords are constructed of thirds and fifths (see Triads), this tuning makes typical Western harmonies particularly resonant and pleasing to the ear; so this tuning is often used (sometimes unconsciously) by musicians who can make small tuning adjustments quickly. This includes vocalists, most wind instruments, and many string instruments.

As explained above, using pure fifths and thirds will require some sort of adjustment somewhere. Just intonation makes two accommodations to allow its pure intervals. One is to allow inequality in the other intervals. Look again at the harmonic series.



Both the 9:8 ratio and the 10:9 ratio in the harmonic series are written as whole notes. 9:8 is considered a **major whole tone** and 10:9 a **minor whole tone**. The difference between them is less than a quarter of a semitone.

As the series goes on, the ratios get smaller and the notes closer together. Common notation writes all of these "close together" intervals as whole steps (whole tones) or half steps (semitones), but they are of course all slightly different from each other. For example, the notes with frequency ratios of 9:8 and 10:9 and 11:10 are all written as whole steps. To compare how close (or far) they actually are, turn the ratios into decimals.

Whole Step Ratios Written as Decimals

- 9/8 = 1.125
- 10/9 = 1.111
- 11/10 = 1.1

These are fairly small differences, but they can still be heard easily by the human ear. Just intonation uses both the 9:8 whole tone, which is called a **major whole tone** and the 10:9 whole tone, which is called a **minor whole tone**, in order to construct both pure thirds and pure fifths.

Note: In case you are curious, the size of the whole tone of the "mean tone" system is also the mean, or average, of the major and minor whole tones.

The other accommodation with reality that just intonation must make is the fact that a single just-intonation tuning cannot be used to play in multiple keys. In constructing a just-intonation tuning, it matters which steps of the scale are major whole tones and which are minor whole tones, so an instrument tuned exactly to play with just intonation in the key of C major will have to retune to play in C sharp major or D major. For instruments that can tune almost instantly, like voices, violins, and trombones, this is not a problem; but it is unworkable for pianos, harps, and other other instruments that cannot make small tuning adjustments quickly.

As of this writing, there was useful information about various tuning systems at several different websites, including The Development of Musical Tuning Systems, where one could hear what some intervals sound like in the different tuning systems, and Kyle Gann's Just Intonation Explained, which included some audio samples of works played using just intonation.

Temperament

There are times when tuning is not much of an issue. When a good choir sings in harmony without instruments, they will tune without even thinking about it. All chords will tend towards pure fifths and thirds, as well as

seconds, fourths, sixths, and sevenths that reflect the harmonic series. Instruments that can bend most pitches enough to fine-tune them during a performance - and this includes most orchestral instruments - also tend to play the "pure" intervals. This can happen unconsciously, or it can be deliberate, as when a conductor asks for an interval to be "expanded" or "contracted".

But for many instruments, such as piano, organ, harp, bells, harpsichord, xylophone - any instrument that cannot be fine-tuned quickly - tuning is a big issue. A harpsichord that has been tuned using the Pythagorean system or just intonation may sound perfectly in tune in one key - C major, for example - and fairly well in tune in a related key - G major - but badly out of tune in a "distant" key like D flat major. Adding split keys or extra keys can help (this was a common solution for a time), but also makes the instrument more difficult to play. In Western music, the tuning systems that have been invented and widely used that directly address this problem are the various temperaments, in which the tuning of notes is "tempered" slightly from pure intervals. (Non-Western music traditions have their own tuning systems, which is too big a subject to address here. See Listening to Balinese Gamelan and Indian Classical Music: Tuning and Ragas for a taste of what's out there.)

Well Temperaments

As mentioned above, the various tuning systems based on pure intervals eventually have to include "wolf" intervals that make some keys unpleasant or even unusable. The various **well temperament** tunings that were very popular in the 18th and 19th centuries tried to strike a balance between staying close to pure intervals and avoiding wolf intervals. A well temperament might have several pure fifths, for example, and several fifths that are smaller than a pure fifth, but not so small that they are "wolf" fifths. In such systems, tuning would be noticeably different in each key, but every key would still be pleasant-sounding and usable. This made well temperaments particularly welcome for players of difficult-to-tune instruments like the harpsichord and piano.

Note: Historically, there has been some confusion as to whether or not well temperament and equal temperament are the same thing, possibly because well temperaments were sometimes referred to at the time as "equal temperament". But these well temperaments made all keys equally useful, not equal-sounding as modern equal temperament does.

As mentioned above, mean-tone tuning was still very popular in the eighteenth century. J. S. Bach wrote his famous "Well-Tempered Klavier" in part as a plea and advertisement to switch to a well temperament system. Various well temperaments did become very popular in the eighteenth and nineteenth centuries, and much of the keyboard-instrument music of those centuries may have been written to take advantage of the tuning characteristics of particular keys in particular well temperaments. Some modern musicians advocate performing such pieces using well temperaments, in order to better understand and appreciate them. It is interesting to note that the different keys in a well temperament tuning were sometimes considered to be aligned with specific colors and emotions. In this way they may have had more in common with various modes and ragas than do keys in equal temperament.

Equal Temperament

In modern times, well temperaments have been replaced by equal temperament, so much so in Western music that equal temperament is considered standard tuning even for voice and for instruments that are more likely to play using just intonation when they can (see above). In equal temperament, only octaves are pure intervals. The octave is divided into twelve equally spaced half steps, and all other intervals are measured in half steps. This gives, for example, a fifth that is a bit smaller than a pure fifth, and a major third that is larger than the pure major third. The differences are smaller than the wolf tones found in other tuning systems, but they are still there.

Equal temperament is well suited to music that changes key often, is very chromatic, or is harmonically complex. It is also the obvious choice for atonal music that steers away from identification with any key or tonality at all. Equal temperament has a clear scientific/mathematical basis, is very straightforward, does not require retuning for key changes, and is unquestioningly accepted by most people. However, because of the lack of pure intervals, some musicians do not find it satisfying. As mentioned above, just intonation is sometimes substituted for equal temperament when practical, and some musicians would also like to reintroduce well temperaments, at least for performances of music which was composed with well temperament in mind.

A Comparison of Equal Temperament with the Harmonic Series

In a way, equal temperament is also a compromise between the Pythagorean approach and the mean-tone approach. Neither the third nor the fifth is pure, but neither of them is terribly far off, either. Because equal temperament divides the octave into twelve equal semi-tones (half steps), the frequency ratio of each semi-tone is the twelfth root of 2. If you do not understand why it is the twelfth root of 2 rather than, say, one twelfth, please see the explanation below. (There is a review of powers and roots in Powers, Roots, and Equal Temperament if you need it.)

$\sqrt[12]{2}$ = a semitone (half step)

$(\sqrt[12]{2})^2$ = a whole tone (whole step)

$(\sqrt[12]{2})^4$ = a major third (four semitones)

$(\sqrt[12]{2})^7$ = a perfect fifth (seven semitones)

$(\sqrt[12]{2})^{12} = 2$ = an octave (twelve semitones)

In equal temperament, the ratio of frequencies in a semitone (half step) is the twelfth root of

two. Every interval is then simply a certain number of semitones. Only the octave (the twelfth power of the twelfth root) is a pure interval.

In equal temperament, the only pure interval is the octave. (The twelfth power of the twelfth root of two is simply two.) All other intervals are given by irrational numbers based on the twelfth root of two, not nice numbers that can be written as a ratio of two small whole numbers. In spite of this, equal temperament works fairly well, because most of the intervals it gives actually fall quite close to the pure intervals. To see that this is so, look at [link]. Equal temperament and pure intervals are calculated as decimals and compared to each other. (You can find these decimals for yourself using a calculator.)

Comparing the Frequency Ratios for Equal Temperament and Pure Harmonic Series

Interval	Equal Temperament Frequency Ratio		Approximate Difference		Harmonic Series Frequency Ratio
Unison	$(\sqrt[12]{2})^{0} \approx$	1.0000	0.0	1.0000 $\approx$	1/1
Minor Second	$(\sqrt[12]{2})^{1} \approx$	1.0595	0.0314	1.0909 $\approx$	12/11
Major Second	$(\sqrt[12]{2})^{2} \approx$	1.1225	0.0025	1.1250 $\approx$	9/8
Minor Third	$(\sqrt[12]{2})^{3} \approx$	1.1892	0.0108	1.2000 $\approx$	6/5
Major Third	$(\sqrt[12]{2})^{4} \approx$	1.2599	0.0099	1.2500 $\approx$	5/4
Perfect Fourth	$(\sqrt[12]{2})^{5} \approx$	1.3348	0.0015	1.3333 $\approx$	4/3
Tritone	$(\sqrt[12]{2})^{6} \approx$	1.4142	0.0142	1.4000 $\approx$	7/5
Perfect Fifth	$(\sqrt[12]{2})^{7} \approx$	1.4983	0.0017	1.5000 $\approx$	3/2
Minor Sixth	$(\sqrt[12]{2})^{8} \approx$	1.5874	0.0126	1.6000 $\approx$	8/5
Major Sixth	$(\sqrt[12]{2})^{9} \approx$	1.6818	0.0151	1.6667 $\approx$	5/3
Minor Seventh	$(\sqrt[12]{2})^{10} \approx$	1.7818	0.0318	1.7500 $\approx$	7/4
Major Seventh	$(\sqrt[12]{2})^{11} \approx$	1.8897	0.0564	1.8333 $\approx$	11/6
Octave	$(\sqrt[12]{2})^{12} \approx$	2.0000	0.0	2.0000 $\approx$	2/1

Look again at [link] to see where pure interval ratios come from. The ratios for

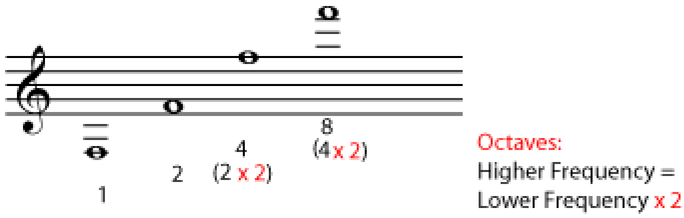
equal temperament are all multiples of the twelfth root of two. Both sets of ratios are converted to decimals (to the nearest ten thousandth), so you can easily compare them.

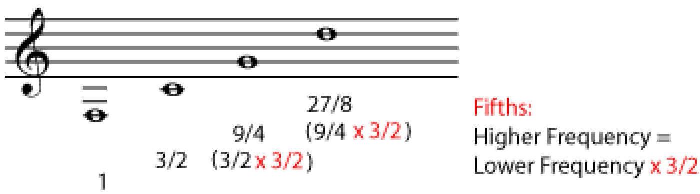
Except for the unison and the octave, none of the ratios for equal temperament are exactly the same as for the pure interval. Many of them are reasonably close, though. In particular, perfect fourths and fifths and major thirds are not too far from the pure intervals. The intervals that are the furthest from the pure intervals are the major seventh, minor seventh, and minor second (intervals that are considered dissonant anyway).

Because equal temperament is now so widely accepted as standard tuning, musicians do not usually even speak of intervals in terms of ratios. Instead, tuning itself is now defined in terms of equal-temperament, with tunings and intervals measured in cents. A **cent** is 1/100 (the hundredth root) of an equal-temperament semitone. In this system, for example, the major whole tone discussed above measures 204 cents, the minor whole tone 182 cents, and a pure fifth is 702 cents.

Why is a cent the hundredth root of a semitone, and why is a semitone the twelfth root of an octave? If it bothers you that the ratios in equal temperament are roots, remember the pure octaves and fifths of the harmonic series.

Frequency Relationships





Remember that, no matter what note you start on, the note one octave higher has 2 times its frequency. Also, no matter what note you start on, the note that is a perfect fifth higher has exactly one and a half times its frequency. Since each of these intervals is so many "times" in terms of frequencies, when you **add** intervals, you **multiply** their frequencies. For example, a series of two perfect fifths will give a frequency that is 3/2 x 3/2 (or 9/4) the beginning frequency.

Every octave has the same frequency ratio; the higher note will have 2 **times** the frequency of the lower note. So if you go up another octave from there (another 2 times), that note must have 2 x 2, or 4 times the frequency of the lowest note. The next octave takes you up 2 times higher than that, or 8 times the frequency of the first note, and so on.

In just the same way, in every perfect fifth, the higher note will have a frequency one and a half (3/2) times the lower note. So to find out how

much higher the frequency is after a series of perfect fifths, you would have to multiply (not add) by one and a half (3/2) every time you went up another perfect fifth.

All intervals work in this same way. So, in order for twelve semitones (half steps) to equal one octave, the size of a half step has to be a number that gives the answer "2" (the size of an octave) when you multiply it twelve times: in other words, the twelfth root of two. And in order for a hundred cents to equal one semitone, the size of a cent must be the number that, when you multiply it 100 times, ends up being the same size as a semitone; in other words, the hundredth root of the twelfth root of two. This is one reason why most musicians prefer to talk in terms of cents and intervals instead of frequencies.

Beats and Wide Tuning

One well-known result of tempered tunings is the aural phenomenon known as **beats**. As mentioned above, in a pure interval the sound waves have frequencies that are related to each other by very simple ratios. Physically speaking, this means that the two smooth waves line up together so well that the combined wave - the wave you hear when the two are played at the same time - is also a smooth and very steady wave. Tunings that are slightly off from the pure interval, however, will result in a combined wave that has an extra bumpiness in it. Because the two waves are each very even, the bump itself is very even and regular, and can be heard as a "beat" - a very regular change in the intensity of the sound. The beats are so regular, in fact, that they can be timed; for equal temperament they are on the order of a beat per second in the mid range of a piano. A piano tuner works by listening to and timing these beats, rather than by being able to "hear" equal temperament intervals precisely.

It should also be noted that some music traditions around the world do not use the type of precision tunings described above, not because they can't, but because of an aesthetic preference for **wide tuning**. In these traditions, the sound of many people playing precisely the same pitch is considered a thin, uninteresting sound; the sound of many people playing near the same pitch is heard as full, lively, and more interesting.

Some music traditions even use an extremely precise version of wide tuning. The gamelan orchestras of southeast Asia, for example, have an aesthetic preference for the "lively and full" sounds that come from instruments playing near, not on, the same pitch. In some types of gamelans, pairs of instruments are tuned very precisely so that each pair produces beats, and the rate of the beats is the same throughout the entire range of that gamelan. Long-standing traditions allow gamelan craftsmen to reliably produce such impressive feats of tuning.

Further Study

As of this writing:

- Kyle Gann's An Introduction to Historical Tunings is a good source about both the historical background and more technical information about various tunings. It also includes some audio examples.
- The Huygens-Fokker Foundation has a very large on-line bibliography of tuning and temperament.
- Alfredo Capurso, a researcher in Italy, has developed the Circular Harmonic System (c.ha.s), a tempered tuning system that solves the wolf fifth problem by adjusting the size of the octave as well as the fifth. It also provides an algorithm for generating microtonal scales. You can read about it at the Circular Harmonic System website or download a paper on the subject. You can also listen to piano performances using this tuning by searching for "CHAS tuning" at YouTube.
- A number of YouTube videos provide comparisons that you can listen to, for example comparisons of just intonation and equal temperament, or comparisons of various temperaments.